LECTURE # 15, Fri., 11-4

→ Angular momentum is a vector; we need to know its direction, or, equivalently, its projection onto a spatial (lab-fixed) axis, conventionally taken to be the z-axis: this projection tells us how the electron behaves in a magnetic field (applied along z)

$m\frac{h}{2\pi}$, the projection of the angular momentum L along the z-axis, takes on the special values

$$L_z = m\frac{h}{2\pi}, \qquad m = -l, -l+1, \ldots, 0, 1, \ldots, l-1, l$$

($2l+1$ values)

$2l+1$ = 1 for $l=0$ (s-state)

3 for $l=1$ (p-states)
($m = -1, 0, +1$)

5 for $l=2$ (d-states)
($m = -2, -1, 0, +1, +2$)

7 for $l=3$ (f-states)
($m = -3, -2, -1, 0, +1, +2, +3$)

etc.

So: $n = 1, 2, 3, \ldots$

l's (for each n) $= 0, 1, \ldots, n-1$ (n)

m's (for each m) $= -l, \ldots, 0, \ldots, +l$ ($2l+1$)

Wavefunctions $\psi_{nlm}(r,\theta,\varphi) = R_{nl}(r)\,Y_{lm}(\theta,\varphi)$ corresponding to these allowed states are shown in Table 15.2, p. 538 (SEE NEXT PAGE)

ABLE 15.2

NGULAR AND RADIAL PARTS OF WAVE FUNCTIONS R ONE-ELECTRON ATOMS

	Angular Part $Y(\theta, \phi)$	Radial Part $R_{n\ell}(r)$
		$R_{1s} = 2\left(\frac{Z}{a_0}\right)^{3/2} \exp(-\sigma)$
$\ell = 0$	$Y_s = \left(\frac{1}{4\pi}\right)^{1/2}$	$R_{2s} = \frac{1}{2\sqrt{2}}\left(\frac{Z}{a_0}\right)^{3/2}(2 - \sigma) \exp(-\sigma/2)$
		$R_{3s} = \frac{2}{81\sqrt{3}}\left(\frac{Z}{a_0}\right)^{3/2}(27 - 18\sigma + 2\sigma^2) \exp(-\sigma/3)$
$\ell = 1$	$Y_{px} = \left(\frac{3}{4\pi}\right)^{1/2} \sin\theta \cos\phi$	$R_{2p} = \frac{1}{2\sqrt{6}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma \exp(-\sigma/2)$
	$Y_{py} = \left(\frac{3}{4\pi}\right)^{1/2} \sin\theta\, \cancel{\cos\phi}\ \sin\phi$	$R_{3p} = \frac{4}{81\sqrt{6}}\left(\frac{Z}{a_0}\right)^{3/2}(6\sigma - \sigma^2) \exp(-\sigma/3)$
	$Y_{pz} = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta$	
2	$Y_{dz^2} = \left(\frac{5}{16\pi}\right)^{1/2} (3\cos^2\theta - 1)$	
	$Y_{dxz} = \left(\frac{15}{4\pi}\right)^{1/2} \sin\theta \cos\theta \cos\phi$	
	$Y_{dyz} = \left(\frac{15}{4\pi}\right)^{1/2} \sin\theta \cos\theta \sin\phi$	$R_{3d} = \frac{4}{81\sqrt{30}}\left(\frac{Z}{a_0}\right)^{3/2} \sigma^2 \exp(-\sigma/3)$
	$Y_{dxy} = \left(\frac{15}{16\pi}\right)^{1/2} \sin\theta$ sib) 2ϕ	
	$Y_{dx^2-y^2} = \left(\frac{15}{16\pi}\right)^{1/2} \sin^2\theta \cos 2\phi$	

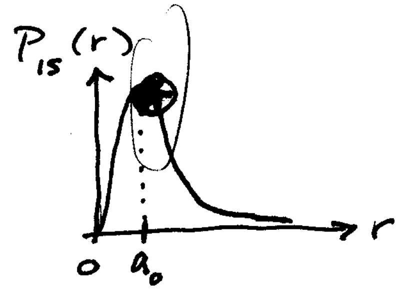
$$\sigma = \frac{Zr}{a_0} \qquad a_0 = \frac{\epsilon_0 h^2}{\pi e^2 m_e} = 0.529 \times 10^{-10}\ \mathrm{m}$$

* BACK TO <u>RADIAL</u> PROB. DENSITIES

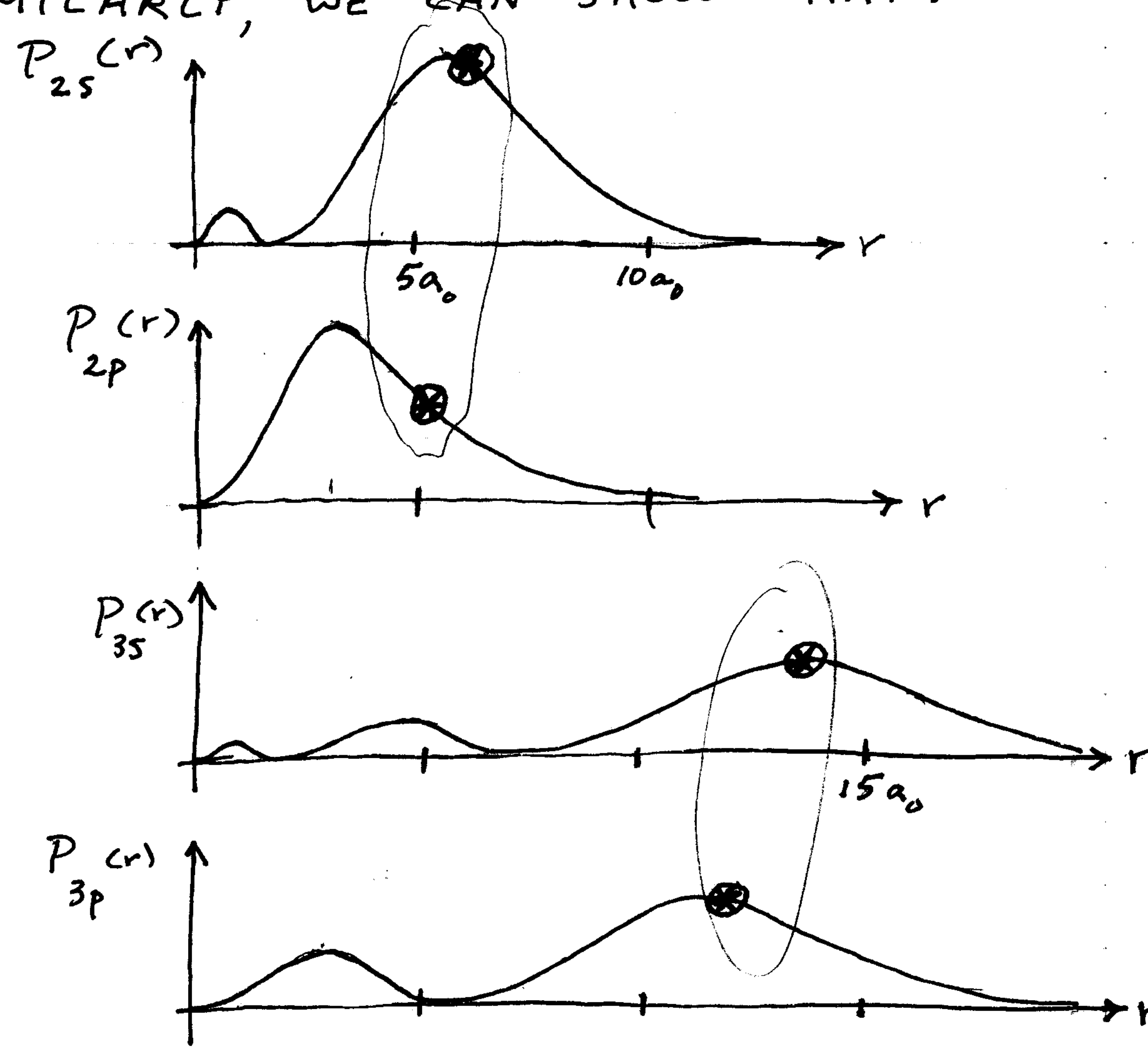
→ For ψ_{1s} we showed that

prob. of finding e between r and r+dr from nucleus $\propto \underbrace{r^2 e^{-2r/a_0}}_{P_{1s}(r)} dr$

with



SIMILARLY, WE CAN SHOW THAT:



⊛ ⟷ average distance $\bar{r}$ from nucleus

FIGURE 15.32 Radial probability densities for hydrogen orbitals with $n = 1, 2, 3$. The small arrow on each curve locates the value of $\bar{r}_{n\ell}$ in that orbital.