LECTURE # 14, Wednes., 11-2

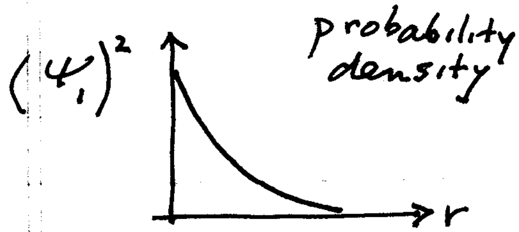
*) Continue discussion from last time:

probability of finding electron ($n=1$) between r and $r+dr$ from nucleus, i.e., inside spherical shell (radius r, thickness dr)

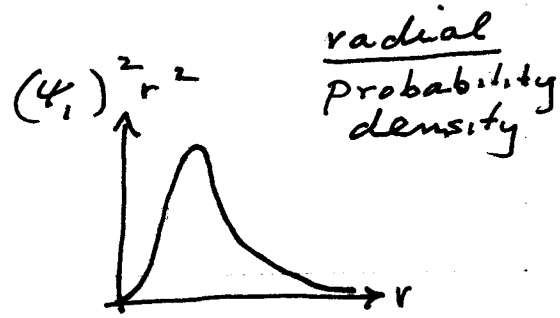
$$= \underbrace{\left(\psi_1(r,\theta,\varphi)\right)^2}_{\frac{1}{\pi}\frac{1}{a_0^3}e^{-2r/a_0}} \underbrace{(\text{volume of shell})}_{4\pi r^2\,dr}$$

$$= \frac{4}{a_0^3}\, e^{-2r/a_0}\, r^2\,dr$$

So:



whereas



*) Wavefunctions for higher-energy states are more complicated

For example

$$E_{n=2} = -\frac{R}{2^2} = -\frac{R}{4} \Longrightarrow 4 \text{ different } \psi_2(r,\theta,\varphi)\text{'s!}$$

$$\psi_{2s}(r,\theta,\varphi) = \left(\frac{1}{4\pi}\right)^{1/2} \frac{1}{2\sqrt{2}} \left(\frac{1}{a_0}\right)^{3/2} \left(2-\frac{r}{a_0}\right) e^{-r/2a_0}$$

$$\psi_{2p_x}(r,\theta,\varphi) = \left(\frac{3}{4\pi}\right)^{1/2} \sin\theta\cos\varphi \;\frac{1}{2\sqrt{6}} \left(\frac{1}{a_0}\right)^{3/2} \frac{r}{a_0} e^{-r/2a_0}$$

NOTE MISTAKE in Table 15.2

$$\psi_{2p_y}(r,\theta,\varphi) = \left(\frac{3}{4\pi}\right)^{1/2} \sin\theta \sin\varphi \quad ''$$

$$\psi_{2p_z}(r,\theta,\varphi) = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta \quad ''$$

For $E_{n=3} = -\frac{R}{3^2} = -\frac{R}{9}$, things are even more complicated: instead of 1 allowed state with this energy (as for E_1), or 3 states (as for E_2), there are ... 9 ! (AND SO ON, FOR $n \geq 4$)

SEE Table 15.2, p. 538

→ Each allowed state corresponds not only to a special energy (E_1, or E_2, or E_3, ...) [$E_n = -R/n^2$, $n = 1, 2, \ldots$] but also to a special (<u>also</u> quantized) angular "mvr" momentum: 0, or $\sqrt{2}\frac{h}{2\pi}$, or $\sqrt{6}\frac{h}{2\pi}$, ... $\sqrt{\ell(\ell+1)}\frac{h}{2\pi}$, $\ell = 0, 1, 2, \ldots$

E.g., $\psi_1 \longleftrightarrow \ell = 0$

$\psi_{2s} \longleftrightarrow \ell = 0$

$\psi_{2p_{x,y,z}} \longleftrightarrow \ell = 1$

For $n = 3$ states, there is one (ψ_{3s}) with $\ell = 0$, 3 ($\psi_{3p_{x,y,z}}$) with $\ell = 1$, and 5 ($\psi_{3d's}$) with $\ell = 2$

→ IN GENERAL, THERE ARE n^2 ALLOWED STATES CORRESPONDING TO THE ENERGY E_n

		$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$
$n^2 = 1$	($E_{n=1}$)	1s			
4	($E_{n=2}$)	2s	+ 3 2p's		
9	($E_{n=3}$)	3s	+ 3 3p's	+ 5 3d's	
16	($E_{n=4}$)	4s	+ 3 4p's	+ 5 4d's	+ 7 4f's