LECTURE # 11, Mon., 10/24

*) Bohr model of H-atom

→ Balance of Coulomb attractive force, $Ze^2/4\pi\epsilon_0 r^2$, and centrifugal outward force, $m_e v^2/r$, AND

→ Quantization of angular momentum, $m_e v r = nh/2\pi$, $n = 1, 2, \ldots$, LEADS TO

$$r_n = \underbrace{\frac{\epsilon_0 h^2}{\pi e^2 m_e}}_{a_0} \frac{n^2}{Z} = \underbrace{a_0}_{= 0.53 \text{Å}} \frac{n^2}{Z}$$

→ From $m_e v r = nh/2\pi$, it follows that

$$v_n = \frac{e^2}{2\epsilon_0 h} \frac{Z}{n}$$

→ Finally, substituting for r_n and v_n in

$$E = -\frac{Ze^2}{4\pi\epsilon_0 r} + \frac{1}{2} m_e v^2$$

$$\Rightarrow \quad E_n = -\underbrace{\boxed{\frac{m_e e^4}{8\epsilon_0^2 h^2}}}_{R} \frac{Z^2}{n^2}, \quad n = 1, 2, 3, \ldots$$

$$R = 2.18 \times 10^{-18}\ J = 1310\ \frac{kJ}{mole}$$

DRAW ON BOARD, AND DISCUSS, E_n - SPECTRUM, i.e., discrete (but infinite!) set of possible energies for bound states of electron

* ionization energy
* absorption and emission "lines"

*) SO FAR, "WE" HAVE <u>QUANTIZED</u>:

→ energies of vibrations in solid (PLANCK)
[to account for spectrum of black-body radiation]

→ energies of EM waves (EINSTEIN)
[to account for photoelectric effect]

→ energies of electrons in H-like atoms (BOHR)
[to account for line spectra of atoms]

<u>WE</u> <u>NEED</u> <u>A</u> <u>REAL</u> <u>THEORY</u> {LESS AD HOC, MORE GENERAL}, rather than a bunch of different hypotheses

This was provided by the "wave equation" theory of Schrödinger, in 1926, which in turn was partially inspired/motivated by the views of de Broglie 1924 on "wave-particle duality"

- More explicitly, de Broglie viewed the electrons in Bohr's orbits (bound states) as <u>standing waves</u> whose wavelengths λ have to be contained an integral number of times in their orbit's circumference, $2\pi r$:

$$n\lambda = 2\pi r$$

- But Bohr's condition on the angular momentum, $m_e v r = nh/2\pi$, <u>also</u> imposed a special (quantization!) condition on the electron orbit's circumference:

$$2\pi r = n\left(\frac{h}{m_e v}\right),$$

THEREBY IMPLYING A FUNDAMENTAL RELATIONSHIP BETWEEN THE ELECTRON'S WAVELENGTH AND MOMENTUM!

$$\boxed{\lambda = \frac{h}{m v} = \frac{h}{p}}$$

→ DE BROGLIE PROPOSED THAT THIS RELATION SHOULD HOLD -- not only for electrons, but also -- FOR ALL PARTICLES, AND FOR LIGHT WAVES!...!

→ A typical e velocity in H-atom is about $1.0 \times 10^{+6}$ m/s. This implies a (de Broglie) wavelength of about 7 Å ($\gg a_0 \approx 0.5$ Å ≈ size of atom)

→ A baseball, with a speed of 70 mi/hr (↔ ≈ 30 m/s) and mass of ⅓ lb (↔ ≈ 0.15 kg), on the other hand, has a de Broglie wavelength of about 1.5×10^{-24} Å ($\ll \cdots \ll 10^{9}$ Å ≈ size of ball $\ll 10^{12}$ Å ≈ size of field)

∴ We don't see the wave-like properties of baseballs (or of any other macroscopic -- e.g., visible to the naked eye -- objects)

[Similarly, we will learn -- from "particle in a box" model, say, that energy differences between quantized energies of baseballs (and other macroscopic systems) are far too small to see (measure)!]

NOTE: The Davisson-Germer experiment, 1927 wavelength properties of electrons are especially apparent, and useful, in the instance of determining crystal structures (particularly, of their surfaces) via e-diffraction (e.g., LEED)