Chemistry 113A
Fall Quarter
Midterm (I), Oct. 28, 2005

Name: Midterm #1 KEY

Student ID #:

Problem 1 (60pts.):

Problem 2 (40pts.):

Problem 3 (60pts.):

Problem 4 (40pts.):

Total Score:

I)
30 pts.

Name 3 scientists that contributed to the development of quantum mechanics and explain their contributions?

Any 3 scientist involved in quantum mechanics received credit.

Including, but not limited to:

Planck
Bohr
Einstein
Chadwick
Rutherford
Thompson ... etc

II)
50 pts.

A) What is the temperature of a blackbody when it is white hot (the wavelength of maximum emission is ~ 500nm).What is the temperature when it is red hot (the wavelength of maximum emission is ~ 600 nm)?
B) What is the reduced mass for a HD molecule, in amu, if the mass of H_2 is 2 amu and the mass of D_2 is 4 amu(atomic mass units)? (1amu=$1.0x^{-27}$ kg)
C) If the wavelength of a free HD molecule is 1.0 nm what is its kinetic energy in electron volts (eV)? 1eV=$1.6x10^{-19}$J.

(A) $\lambda_1 T_1 = \lambda_2 T_2$

$$T_2 = \frac{\lambda_1 T_1}{\lambda_2} = \frac{(5000\,K)(500\,nm)}{600\,nm}$$

$$= 4166\ K$$

(B) $$\mu = \frac{m_1 m_2}{m_1 + m_2} \to \frac{(1)(2)}{(1)+(2)} = \frac{2}{3}\ amu$$

(C) $$p = \frac{h}{\lambda} = \frac{6.626E^{-34}\ J \cdot s}{1.0E^{-9}\ m} = 6.626E^{-25}\ kg\frac{m}{s}$$

$$KE = \frac{p^2}{2\mu} = \frac{\left(6.626E^{-25}\ kg\frac{m}{s}\right)^2}{2\left(\frac{2}{3}(1.0\times10^{-27}kg)\right)} = 3.29E^{-22}\ Joules$$

$$\Rightarrow 0.002\ eV$$

III)
60 pts.

If a system has eigenfunctions $\psi_n(y)$ with energy eigenvalues $E_n=E_1/n^2$, n= all positive integers.

A) If the system was in a state $\phi(x)= \psi_1(y)+ 0.5\psi_2(y)+ 0.707\psi_3(y)$, what is the probability of finding the system with energy exactly equal to $E_1/4$?

B) What is the average energy of the system in terms of E_1?

C) Show that $\phi(x)$ is, or is not, a solution of the time independent Schrödinger Equation, assume a mass μ.

$\Rightarrow \langle \phi | \phi \rangle = \int \phi^* \phi \, d\tau$

(A)

$$P_2 = \frac{|\langle \psi_2 | \phi \rangle|^2}{\langle \phi | \phi \rangle} = \frac{\langle \phi | \psi_2 \rangle \langle \psi_2 | \phi \rangle}{\langle \phi | \phi \rangle}$$

$$= \frac{\langle \psi_1 + 0.5\psi_2 + 0.7\psi_3 | \psi_2 \rangle \langle \psi_2 | \psi_1 + 0.5\psi_2 + 0.7\psi_3 \rangle}{\langle \psi_1 + 0.5\psi_2 + 0.7\psi_3 | \psi_1 + 0.5\psi_2 + 0.7\psi_3 \rangle}$$

$$= \frac{\left(\langle \psi_1 | \psi_2 \rangle^{0} + 0.5\langle \psi_2 | \psi_2 \rangle^{1} + 0.7\langle \psi_3 | \psi_2 \rangle^{0}\right)\left(\langle \psi_2 | \psi_1 \rangle^{0} + 0.5\langle \psi_2 | \psi_2 \rangle^{1} + 0.7\langle \psi_2 | \psi_3 \rangle^{0}\right)}{\begin{gathered}\langle \psi_1 | \psi_1 \rangle^{1} + 0.5\langle \psi_1 | \psi_2 \rangle^{0} + 0.7\langle \psi_1 | \psi_3 \rangle^{0} + 0.5\langle \psi_2 | \psi_1 \rangle^{0} + 0.5^2\langle \psi_2 | \psi_2 \rangle^{1} \\ + (0.5)(0.7)\langle \psi_2 | \psi_3 \rangle^{0} + 0.7\langle \psi_3 | \psi_1 \rangle^{0} + (0.7)(0.5)\langle \psi_3 | \psi_2 \rangle^{0} \\ + (0.7)^2\langle \psi_3 | \psi_3 \rangle^{1}\end{gathered}}$$

$$= \frac{(0.5)(0.5)}{1 + 0.5^2 + 0.7^2} = 0.143$$

$$\Rightarrow 14.3\%$$

(B)

$$\langle E \rangle = \frac{\langle \phi | \hat{H} | \phi \rangle}{\langle \phi | \phi \rangle} = \frac{(1)^2 E_1 + (0.5)^2 \frac{E_1}{4} + (0.7)^2 \frac{E_1}{9}}{1^2 + 0.5^2 + 0.7^2}$$

because $\Rightarrow$ ϕ isn't normalized

(c) $\hat{H}\alpha = \hat{H}(\psi_1 + 0.5\psi_2 + 0.7\psi_3) = \hat{H}\psi_1 + 0.5\hat{H}\psi_2 + 0.7\hat{H}\psi_3$

$= E_1\psi_1 + 0.5E_2\psi_2 + 0.7E_3\psi_3$

$= E_1\psi_1 + \frac{0.5E_1}{4} + \frac{0.7}{9}E_1\psi_3$

$\neq E_T\alpha$

IV)
60 pts.

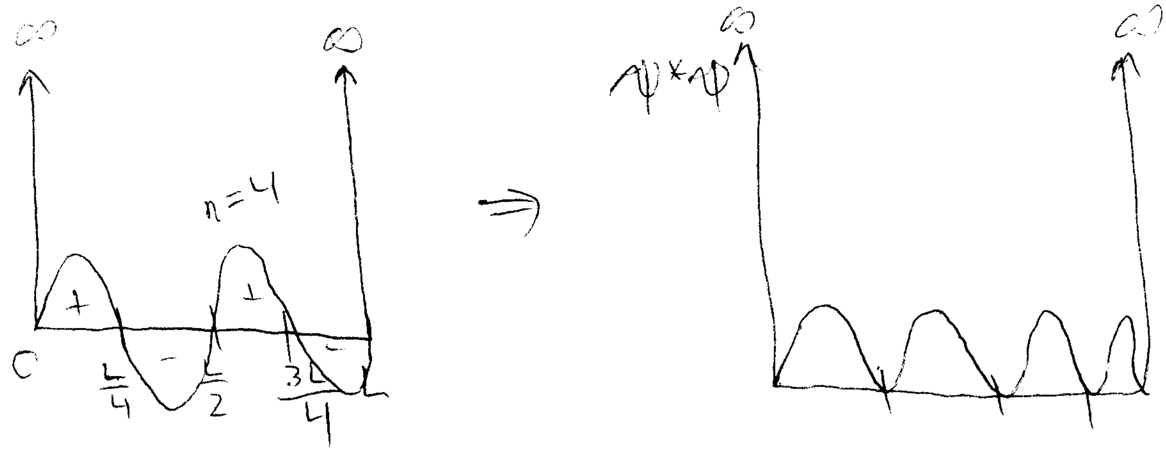
A system of reduced mass μ is bounded by an infinite square well potential in a region of width L.

A) Plot the potential and sketch the eigenfunction and probability density for the 3rd excited state in the region of the potential. $n = 4$

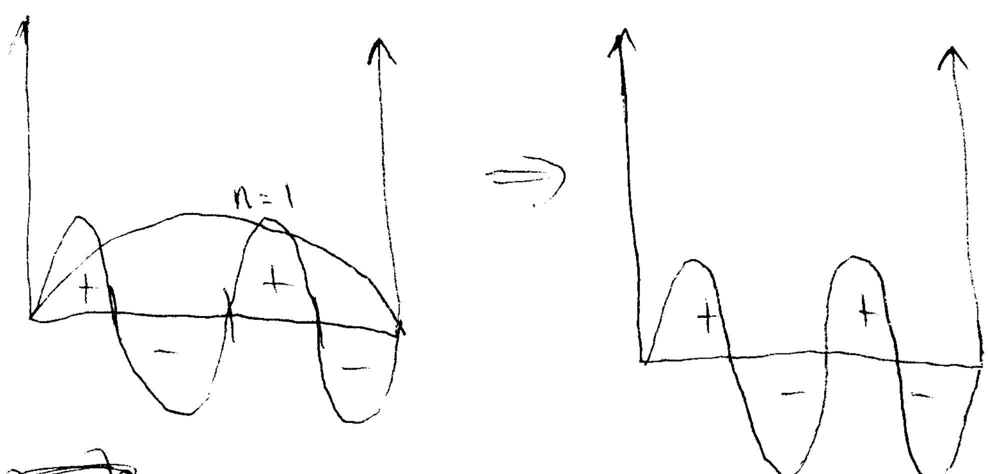
B) Demonstrate graphically the orthogonality between the eigenfunction in part A) and the ground state eigenfunction. Space coordinates on the plots should be given in terms of the length L (i.e., L/3, L/4 etc.).

C) Using the concept of de Broglie waves explain why only certain discrete energies occur for this system.

(A)



(B)



positive area cancels negative area

(C) ~~$p = \lambda$~~

~~de Broglie~~ Only certain wavelength will fit into the box, thus only certain momentums are allowed and thus energy in quantized.

$$\lambda_n = \frac{2L}{n} \Rightarrow p_n = \frac{h}{\lambda_n} \Rightarrow E_n = \frac{p_n^2}{2m} = \boxed{\frac{h^2 n^2}{8mL^2}}$$