Cesar Mu

HW #4 KEY

① ⓐ $\mu = 0.5 \times 10^{-30}$ kg $\quad M = 1.0 \times 10^{-30}$ kg

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad M = m_1 + m_2$$

$$\Rightarrow m_1 = M - m_2$$

$$\Rightarrow \mu = \frac{(M - m_2) m_2}{M}$$

$$\Rightarrow M\mu = M m_2 - m_2^2$$

$$m_2^2 - M m_2 + M\mu = 0$$

$$m_2 = \frac{+M \pm \sqrt{M^2 - 4M}}{2}$$

since $M^2 - 4M < 0$ this equation has no real roots...

ⓑ

$$\Psi(x_1, x_2) = \psi_1(x_1)\psi(x_2) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x_1}{L}\right) \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x_2}{L}\right)$$

ⓒ Wavefuction?: Of course...

Eigenfunction? $\hat{H} = \hat{H}_1 + \hat{H}_2 = -\frac{\hbar^2}{2m_1}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m_2}\frac{\partial^2}{\partial x_2^2}$

$$\hat{H}\Psi(x_1, x_2) = (\hat{H}_1 + \hat{H}_2)(\psi_1(x_1)\psi_2(x_2))$$
$$= \psi_2(x_2)\hat{H}_1\psi_1(x_1) + \psi_1(x_1)\hat{H}_2\psi_2(x_2)$$
$$= E_1\psi_2(x_2)\psi_1(x_1) + E_2\psi_1(x_1)\psi_2(x_2)$$
$$= (E_1 + E_2)\psi_1(x_1)\psi_2(x_2)$$
$$= E_{TOTAL}\Psi(x_1, x_2)$$

ⓓ

$$E_{TOTAL} = E_1 + E_2 = \frac{\pi^2\hbar^2}{2mL^2} + \frac{4\pi^2\hbar^2}{2mL^2} = 822.1\text{eV}$$

② For a free particle, let $V(x) = 0$

$$H\psi = E\psi$$

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right)\psi = E\psi$$

$$\frac{\partial^2}{\partial x^2}\psi = -\frac{2mE}{\hbar^2}\psi$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\Rightarrow \psi(x) = Ae^{ikx} + Be^{ikx}$$

Let's simply use $\psi(x) = Ae^{ikx}$.

$$\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$$

$$\hat{p}_x\psi(x) = -i\hbar\frac{\partial}{\partial x}\left(Ae^{ikx}\right) = -i\hbar(ik)Ae^{ikx}$$

$$= \hbar k\, Ae^{ikx}$$

$$= \hbar k\, \psi(x)$$

$$= p\psi(x)$$

$$\Rightarrow p = \hbar k$$

$$\cancel{E = \frac{p^2}{2m} \leftarrow \hbar^2 k^2}$$

$$\cancel{\therefore E = \frac{\hbar^2 k^2}{2m} = \frac{h^2}{2mL^2}}$$

$$\cancel{\therefore p = \sqrt{2mE} = \hbar k}$$

$$p = \hbar\sqrt{\frac{2mE}{\hbar^2}} = \sqrt{2mE}$$

$$\boxed{p = \frac{h}{L}}$$

③ $\hat{A}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\quad |\alpha\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad |\beta\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Clearly, $\langle\alpha|\beta\rangle = 0$ and $\langle\alpha|\alpha\rangle = 1 \Rightarrow \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$

$\langle\beta|\beta\rangle = 1 \quad \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$

row vector → $\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$

column vector

So the basis is orthonormal

$$A_z|\alpha\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad ✓$$

$$A_z|\beta\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -1\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad ✓$$