HW #2 Key (old)

①

$KE = h\nu - \Phi$

$\Phi \Rightarrow$ binding energy $\Rightarrow$ Rydberg formula $\Rightarrow \Phi = \frac{+\mathbb{R}}{n^2} = \frac{+13.6\,eV}{1}$

where $n=1$

$E = h\nu = (6.626E{-}34\,J\cdot s)\left(\frac{3.0E^{8}\,m/s}{90\times10^{-9}\,m}\right)$

$\hookrightarrow \frac{c}{\lambda} \rightarrow = 2.21\times10^{-18}\,J$

$KE = (2.21\times10^{-18}\,J) - 13.6\,eV\left(\frac{1.602\times10^{-19}\,J}{1\,eV}\right)$

$= 3.13\times10^{-20}\,J$

② $\lambda = \frac{h}{p}$

$KE = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mKE} = \sqrt{2(9.11\times10^{-27}\,kg)(3.13\times10^{-20}\,J)}$

$= 2.39\times10^{-23}\,m$

$\lambda = \frac{h}{p} = \frac{6.626E{-}34\,J\cdot s}{2.39\times10^{-23}\,kg\cdot\frac{m}{s}} = 2.77\times10^{-11}\,m$

$= .277\,Å$

③ $\mathbb{R} = 13.6\,eV$

$E_{2\to1} = -\mathbb{R}\left(\frac{1}{2^2} - \frac{1}{1^2}\right)$

$= 10.2\,eV$

~~$\Rightarrow \nu_{2\to1} =$~~

$E_{3\to1} = -\mathbb{R}\left(\frac{1}{3^2} - \frac{1}{1^2}\right)$

$= 12.1\,eV$

$E_{\cancel{3}\,4\to1} = -\mathbb{R}\left(\frac{1}{4^2} - \frac{1}{1^2}\right)$

$= 12.75\,eV$

Use $\lambda = \frac{hc}{E}$ ~~to~~ to get wavelengths

$\Rightarrow$ UV region $\Leftrightarrow$ Lyman Series

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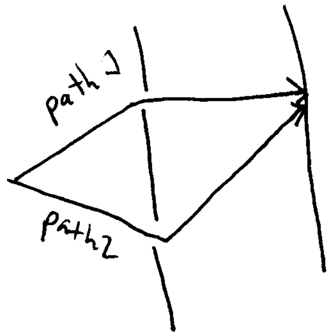


$$\Psi(x) = f(x) + g(x) = \frac{e^{-i\pi x/a}}{\sqrt{2}} + \frac{e^{-i\pi x/2a}}{\sqrt{2}}$$

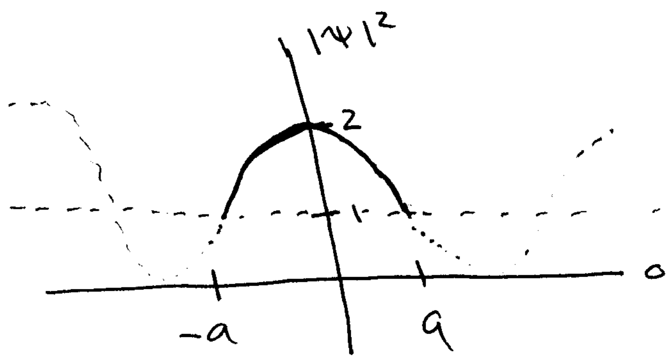
~~P(x)~~ Probability density $= |\Psi(x)|^2 = \Psi^*(x)\Psi(x) = (f^* + g^*)(f+g)$

$$= f^*f + g^*g + f^*g + g^*f$$

$$\cancel{\neq \left(\frac{e^{-i\pi x/a}}{}\right.}$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\left(e^{i\frac{\pi x}{2a}} + e^{-i\frac{\pi x}{2a}}\right)$$

$$= 1 + \cancel{\tfrac{2}{2}}\cos\left(\frac{\pi x}{2a}\right);$$ Recall $\cos x = \frac{1}{2}(e^{ix} + e^{-ix})$



Is $\Psi(x)$ normalized?

$$\int_{-a}^{a} \Psi^*(x)\Psi(x) = 1$$

$$\int_{-a}^{a}\left(1 + \cos\left(\frac{\pi x}{2a}\right)\right)dx = x\Big|_{-a}^{a} + \frac{2a}{\pi}\sin\left(\frac{\pi x}{2a}\right)\Big|_{-a}^{a}$$

$$= 2a + \left(\frac{2a}{\pi} + \frac{2a}{\pi}\right) = \frac{2a\pi + 4a}{\pi} = 1$$

$$\Rightarrow a(2\pi + 4) = \pi$$

$\Psi(x)$ is normalize only for $\Longrightarrow$ $\Rightarrow a = \dfrac{\pi}{2\pi + 4}$

(5) You are more likely to find the particle near $x=0$ since that is where $|\psi(x)|^2$ is maximal.

(6) notice the first emission ~~z~~ series is converging to $E_{1st\text{-}series} = 54\,eV$ and the second series is converging to $E_{2nd} \cong 460\,eV$.

The energy of convergence corresponds to the energy needed to ~~excite~~ ionize the electron.
The ionization energy is the negative of the electron ground state energy given by the Rydberg equation.

atomic number

$n=1$ for ground state

1st Series

$$IE = 54\,eV$$

$$\Rightarrow E_1 = -54\,eV = -R\left(\frac{Z^2}{n^2}\right) = -RZ^2$$

ground state energy of electron

$$\Rightarrow Z = \sqrt{\frac{54\,eV}{13.6}} \approx \sqrt{4} \approx 2$$

$$\boxed{Z=2 \Rightarrow He^{1+}}$$

2nd Series

Similarly,

$$IE = 460\,eV$$

$$\Rightarrow Z=6 \Rightarrow \boxed{C^{5+}}$$