HW#6 KEY

① Pauli Spin Matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{S}_i = \frac{\hbar}{2}\sigma_i$$

$$\hat{S} = (\hat{S}_x, \hat{S}_y, \hat{S}_z) = \hat{S}_x\,\hat{i} + \hat{S}_y\,\hat{j} + \hat{S}_z\,\hat{z}$$

$$\Rightarrow \hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$\Rightarrow \hat{S} = \begin{pmatrix} 0 & \hbar/2 \\ \hbar/2 & 0 \end{pmatrix}\hat{i} + \begin{pmatrix} 0 & -i\hbar/2 \\ i\hbar/2 & 0 \end{pmatrix}\hat{j} + \begin{pmatrix} \hbar/2 & 0 \\ 0 & -\hbar/2 \end{pmatrix}\hat{k}$$

② $|\psi\rangle = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$$[\hat{S}_x, \hat{S}_y]|\psi\rangle = (\hat{S}_x\hat{S}_y - \hat{S}_y\hat{S}_x)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \frac{\hbar^2}{4}\left[\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}\right]\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{\hbar^2}{4}\left[\begin{pmatrix} +2i & 0 \\ 0 & -2i \end{pmatrix}\right]\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$[S_x, S_y] = i\hbar S_z$

$$= \frac{\hbar^2}{2}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{i\hbar^2}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= i\hbar S_z \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

3. The implication of 2 is that you cannot simultaneously ~~measuring the com~~ measure any two components of the electron spin with arbitrary accuracy. Thus, $\hat{S}_x$ and $\hat{S}_y$ do not share a common set of eigenfunctions

4.

$$\langle \psi_1 \alpha | \psi_1 \beta \rangle = \cancelto{1}{\langle \psi_1 | \psi_1 \rangle} \cancelto{0}{\langle \alpha | \beta \rangle} = 0$$

Thus they are ~~orthonormal~~ orthogonal

$$\langle \psi_1 \alpha | \psi_1 \alpha \rangle = \cancelto{1}{\langle \psi_1 | \psi_1 \rangle} \cancelto{1}{\langle \alpha | \alpha \rangle} = 1$$

$$\langle \psi_1 \beta | \psi_1 \beta \rangle = \cancelto{1}{\langle \psi_1 | \psi_1 \rangle} \cancelto{1}{\langle \beta | \beta \rangle} = 1$$

and orthonormal.

5.

$$\underline{\underline{X}} = \begin{bmatrix} \langle 0|\hat{x}|0\rangle & \langle 0|\hat{x}|1\rangle \\ \langle 1|\hat{x}|0\rangle & \langle 1|\hat{x}|0\rangle \end{bmatrix} = \begin{bmatrix} 0 & \frac{\pi}{2}\sqrt{\frac{2}{\alpha}} \\ \frac{\pi}{2}\sqrt{\frac{2}{\alpha}} & 0 \end{bmatrix}$$

6. Transitions from $n=1$ to $n=0$ are allowed and vice-versa. While transitions from $n=0$ to $n=0$ or $n=2$ to $n=2$ are not allowed.